POST FAILURE DEFLECTIONS OF CYLINDRICAL SHELLS UNDER DYNAMIC LATERAL LOADS

Joshua E. Greenspon, Dr. Eng.

Ballistic Research Laboratories
Aberdeen Proving Ground
Contract No. DA-36-034-ORD-3081X
Technical Report No. 5
August, 1964

J G ENGINEERING RESEARCH ASSOCIATES
3831 MENLO DRIVE BALTIMORE 15, MARYLAND

ABSTRACT

The general work-energy approach for calculating plastic deformations is treated . Curves are presented for energy absorbed by cylindrical shells as a function of post failure deflection for both collapse and buckling failure. A set of curves is also given for predicting whether a shell will collapse or buckle first.

TABLE OF CONTENTS

ACKNOWLEDGEMENTS

This work was done under the sponsorship of Ballistic Research Laboratories at Aberdeen Proving Ground. The technical supervisors of the project are Mr. O. T. Johnson and Mr. W. Schuman. The author is grateful to Messrs. Johnson and Schuman for many valuable discussions which he had with them during the progress of this work, and he is grateful to Ballistic Research Laboratories for their continued support. The numerical evaluation of the integrals was programmed for the BRL computer by Mr. Clyde Hauff of the computing laboratory at Aberdeen. His input to this work deserves great recognition.

LIST OF SYMBOLS

p_0, p_1	pressure parameters defining distribution of pressure such that $p_0 + p_1$ is the pressure at $\varphi = 0$
$\bar{\rho}$	$p_1/(p_0 + p_1)$
a	radius of shell
D	diameter of shell
$N_x, N_\varphi, N_{x\varphi}$	membrane force resultants
x, φ	cylindrical coordinates defining coordinates of any point on surface of shell from origin located at center of shell at the point of maximum pressure
ν	Poisson"s ratio for shell material
L	length of shell
p	$p_0 + p_1$
t	thickness of shell
σ_s	yield stress in pure tension
w_0	maximum deflection at center section of shell
x'	x/L
n	number of full waves around the circumference in the buckling pattern
$f(x', \varphi)$	shape of the deflection pattern
d_0	length of hinge in collapse pattern
k	coefficient of exponential describing spatial distribution of post failure buckling pattern
ρ	mass density of shell material
I_t	total impulse
$g(x, \varphi)$	function describing spatial distribution of impulse
p_0	maximum overpressure of blast load
θ	decay time of blast load

c velocity of sound in air

ρ_o density behind shock wave

$\bar{E}$ energy per unit area of plane shock wave

V energy absorbed by shell in plastic deformation

$\bar{V}$ $V / \left(\frac{\sigma_s t a L}{\sqrt{3}} \right)$

$\bar{\alpha}, \bar{\beta}, \bar{\gamma}$ function describing the integrand of the energy integrals

I. Introduction

Two previous reports[1,2*] developed the theory of the elastic and plastic behavior of cylindrical shells subjected to dynamic lateral loads. Due to the complexity of this problem a work-energy type of approach was followed. This approach has been used by several investigators[3,4] in studying the plastic behavior of multiweb beams and ship plating. It is the purpose of this report to present more extensive results for cylindrical shells over a wide range of geometrical paramemters.

II. Behavior of cylindrical shells

A. General

As discussed in earlier work,[2] shells subjected to side on blast can go into two main types of faliure -- buckling or collapse. Extensive experimental resluts showing these types of failure are given by Schuman.[5,6] Approximate relations giving the load at which the shell would collapse or buckle were given in the above mentioned reference.[2] The important point is that if the static buckling load is less than the static yield load the shell should buckle; if the yield load is less than the buckling load the shell should collapse. Once the general pattern of either collapse or buckling is determined, then the post failure analysis using this pattern follows in a straightforward manner by computing the energy integrals numerically.

B. Buckling

A rather extensive study of elastic buckling of a cylindrical shell under nonuniform lateral pressure is given by Almroth.[7] He represents the pressure on the shell by the relation

$$p(\varphi) = p_0 + p_1 \cos\varphi \qquad [1]$$

The pressure at $\varphi = 0$ is $(p_0 + p_1)$. Almroth obtains the critical value of $(p_0 + p_1)$ for buckling of the shell. The nonuniformity of the pressure is described by a parameter $\bar{\rho}$ defined as

$$\bar{\rho} = p_1 / (p_0 + p_1) \qquad [2]$$

He obtains a variety of critical load curves for $\rho = 0.5$; this is the case which will be considered here. These buckling curves are shown as solid lines in Fig. 1. The yield load corresponding to a nonuniform pressure as given above with $\rho = 0.5$ will be considered next.

C. Yield load (corresponding to Almroth's curves of $\rho = 0.5$)

As in a previous reference[2] it will be assumed that the membrane theory holds. The pressure is given by [1] which can also be written

---

* Superscripts refer to references listed at the back of the report

$$p = p_0 f(\varphi) \qquad f(\varphi) = \left(1 + \frac{p_1}{p_0}\cos\varphi\right) \tag{3}$$

The membrane force resultants are then given by

$$\begin{aligned} N_\varphi &= a\, p_0 f(\varphi) \\ N_{x\varphi} &= -p_0\, x\, f'(\varphi) \\ N_x &= \frac{p_0}{a}\frac{x^2}{2} f''(\varphi) + \nu a p_0 f(\varphi) - \frac{p_0}{a}\frac{L^2}{24} f''(\varphi) \end{aligned} \tag{4}$$

in which

$$f'(\varphi) = -\frac{p_1}{p_0}\sin\varphi, \quad f''(\varphi) = -\frac{p_1}{p_0}\cos\varphi$$

So at the center of the shell $x = 0,\ \varphi = 0$

$$\begin{aligned} N_\varphi &= a(p_0 + p_1) \\ N_x &= \nu a(p_0+p_1) + \frac{L^2}{24a} p_1 = \nu a(p_0+p_1) + \frac{L^2}{24}(p_0+p_1)\bar\beta \\ N_{x\varphi} &= 0 \end{aligned} \tag{5}$$

Now let $p = (p_0 + p_1)$ (the critical yield pressure at $\varphi = 0$ corresponds to Almroth's calculations for buckling) [6]

then

$$\begin{aligned} N_\varphi &= a p \\ N_x &= \nu a p + \frac{L^2}{24} p \beta \end{aligned} \tag{7}$$

Using the VonMises yield condition

$$N_x^2 - N_x N_\varphi + N_\varphi^2 + 3 N_{x\varphi}^2 = \sigma_0^2 t^2 \tag{8}$$

the critical pressure for yield is

$$(p)_c = \sigma_s \frac{t}{a} \frac{1}{\sqrt{(1-\nu+\nu^2) + \bar\beta\left(L^2/24a^2\right)(2\nu-1) + \bar\beta^2\left(L^2/24a^2\right)^2}} \tag{9}$$

which for $\beta = 0.5,\ \nu = .3$ reduces to

$$(p)_c = \sigma_s \frac{2}{D/t} \frac{1}{\sqrt{.79 - .03328(L/D)^2 + .00696(L/D)^4}} \tag{10}$$

These yield curves are shown as dotted lines in Figure 1.

Post failure

The complete expression for the elastic and plastic energy absorbed by the shell as a function of deflection is given in the Errata and Addendum section of Reference 1. However it is difficult to ascertain which parts of the shell are elastic and which are plastic at any given state of deformation. Therefore we will limit the analysis to a rigid plastic material neglecting all elastic deformations and assuming that all the energy goes into plastically deforming the shell. If we further limit our analysis to a perfectly plastic material, the absorbed energy can be written[1,8]

$$V = \sigma_s \int_0^L \int_0^{2\pi} \int_{-t/2}^{t/2} e_i \, a \, d\varphi \, dx \, dz \qquad [11]$$

where $\quad e_i = 2/\sqrt{3} \, \sqrt{\epsilon_x^2 + \epsilon_x \epsilon_y + \epsilon_y^2 + \frac{1}{4}\gamma_{xy}^2}$

Using only lateral deflections as explained in Reference 1, the plastic energy absorbed can be written

$$\bar{V} = \frac{V}{\frac{\sigma_s t a L}{\sqrt{3}}} = \int_0^1 \int_0^{2\pi} \left\{ \left[\frac{(2\bar{\beta}+\bar{\gamma})\sqrt{\bar{\alpha}+\bar{\gamma}+\bar{\beta}}}{4\bar{\beta}} + \frac{(4\bar{\alpha}\bar{\beta}-\bar{\gamma}^2)}{8\bar{\beta}\sqrt{\bar{\beta}}} \sinh^{-1}\left(\frac{2\bar{\beta}+\bar{\gamma}}{\sqrt{4\bar{\alpha}\bar{\beta}-\bar{\gamma}^2}}\right) \right. \right. \qquad [12]$$
$$\left. \left. - \left[\frac{(-2\bar{\beta}+\bar{\gamma})\sqrt{\bar{\alpha}-\bar{\gamma}+\bar{\beta}}}{4\bar{\beta}} + \frac{(4\bar{\alpha}\bar{\beta}-\bar{\gamma}^2)}{8\bar{\beta}\sqrt{\bar{\beta}}} \sinh^{-1}\left(\frac{-2\bar{\beta}+\bar{\gamma}}{\sqrt{4\bar{\alpha}\bar{\beta}-\bar{\gamma}^2}}\right) \right. \right. dx' \, d\varphi$$

where

$$\bar{\alpha}(x',\varphi) = \left(\frac{w_0}{a}\right)^4 \left(\frac{a}{L}\right)^4 \frac{1}{4}\left(\frac{\partial f}{\partial x'}\right)^4 + \left(\frac{w_0}{a}\right)^4 \left(\frac{a}{L}\right)^2 \frac{1}{2}\left(\frac{\partial f}{\partial x'}\right)^2 \left(\frac{\partial f}{\partial \varphi}\right)^2 - \nu\left(\frac{w_0}{a}\right)^3 \left(\frac{a}{L}\right)^2 f \left(\frac{\partial f}{\partial x'}\right)^2 \qquad [13]$$
$$+ \frac{1}{4}\left(\frac{w_0}{a}\right)^4 \left(\frac{\partial f}{\partial \varphi}\right)^4 - \left(\frac{w_0}{a}\right)^3 f \left(\frac{\partial f}{\partial \varphi}\right)^2 + \left(\frac{w_0}{a}\right)^2 f^2$$

$$\bar{\gamma}(x',\varphi) = \frac{t}{2}\gamma = -\left(\frac{w_0}{a}\right)^3 \left(\frac{a}{L}\right)^4 \frac{t}{D}\left(\frac{\partial f}{\partial x'}\right)^2 \left(\frac{\partial^2 f}{\partial x'^2}\right) - \left(\frac{w_0}{a}\right)^3 \frac{t}{D}\left(\frac{\partial^2 f}{\partial \varphi^2}\right)\left(\frac{\partial f}{\partial \varphi}\right)^2$$
$$+ 2\left(\frac{w_0}{a}\right)^2 \frac{t}{D} \frac{\partial^2 f}{\partial \varphi^2} f - \nu\left(\frac{w_0}{a}\right)^3 \left(\frac{a}{L}\right)^2 \frac{t}{D}\left(\frac{\partial f}{\partial x'}\right)^2 \left(\frac{\partial^2 f}{\partial \varphi^2}\right)$$
$$- \nu\left(\frac{w_0}{a}\right)^3 \left(\frac{a}{L}\right)^2 \frac{t}{D}\left(\frac{\partial^2 f}{\partial x'^2}\right)\left(\frac{\partial f}{\partial \varphi}\right)^2 + 2\nu\left(\frac{w_0}{a}\right)^2 \left(\frac{a}{L}\right)^2 \frac{t}{D} f \left(\frac{\partial^2 f}{\partial x'^2}\right)$$
$$- \frac{4(1-\nu)}{2}\left(\frac{w_0}{a}\right)^3 \left(\frac{a}{L}\right)^2 \frac{t}{D}\left(\frac{\partial^2 f}{\partial x' \partial \varphi}\right)\left(\frac{\partial f}{\partial x'}\right)\left(\frac{\partial f}{\partial \varphi}\right)$$

$$\bar{\beta}(x',\varphi) = \left(\frac{t}{2}\right)^2 \beta = \left(\frac{w_0}{a}\right)^2 \left(\frac{a}{L}\right)^4 \left(\frac{t}{D}\right)^2 \left(\frac{\partial^2 f}{\partial x'^2}\right)^2 + 2\nu\left(\frac{w_0}{a}\right)^2 \left(\frac{t}{D}\right)^2 \left(\frac{a}{L}\right)^2 \left(\frac{\partial^2 f}{\partial x'^2}\right)\left(\frac{\partial^2 f}{\partial \varphi^2}\right)$$
$$+ \left(\frac{w_0}{a}\right)^2 \left(\frac{t}{D}\right)^2 \left(\frac{\partial^2 f}{\partial \varphi^2}\right)^2 + 2(1-\nu)\left(\frac{w_0}{a}\right)^2 \left(\frac{a}{L}\right)^2 \left(\frac{t}{D}\right)^2 \left(\frac{\partial^2 f}{\partial x' \partial \varphi}\right)^2$$

In the above expression the deflection was assumed to have the form

$$w = w_0 \, f(x', \varphi) \quad ; \quad x' = x/L \qquad [14]$$

The patterns of deformation used were[2]

For buckling

$$w = w_0 \sin \pi x' \, e^{-k\varphi} \cos n\varphi \qquad 0 < \varphi < \pi \qquad [15]$$
$$= w_0 \sin \pi x' \, e^{-k(2\pi-\varphi)} \cos n(2\pi-\varphi) \qquad \pi < \varphi < 2\pi$$

For collapse

$$w = a\cos\varphi - \sqrt{a^2 - \frac{d_0^2}{4}(1-2x')^2} \qquad 0<x'<\tfrac{1}{2}$$

$$= a\cos\varphi - \sqrt{a^2 - \frac{d_0^2}{4}(1+2x')^2} \qquad -\tfrac{1}{2}<x'<0 \qquad [16]$$

The curves of $\bar{V}$ are plotted in Figure 2 for buckling and collapse failure as a function of the deflection and the geometric parameters of the shell. These curves were computed for the parameters $\lambda = 1$, $\nu = \frac{1}{2}$, $k = \frac{1}{4}$

For buckling it was seen in Reference 2 that the n for buckling approximately satisfies the relation

$$\frac{1}{1 + n^2L^2/\pi^2a^2} \simeq 1.23\sqrt{at}/L \qquad [17]$$

The values of n satisfying the above relation for a given D/t, L/D are as follows:

$D/t = 100$	$L/D = 1$	$n = 5$
	$= 2$	$= 4$
	$= 4$	$= 3$
	$= 8$	$= 2$
	$= 12$	$= 2$
$D/t = 200$	$= 1$	$= 6$
	$= 2$	$= 4$
	$= 4$	$= 3$
	$= 8$	$= 2$
	$= 12$	$= 2$
$D/t = 400$	$= 1$	$= 7$
	$= 2$	$= 5$
	$= 4$	$= 4$
	$= 8$	$= 3$
	$= 12$	$= 2$
$D/t = 800$	$= 1$	$= 9$
	$= 2$	$= 6$
	$= 4$	$= 5$
	$= 8$	$= 3$
	$= 12$	$= 2$

$D/t = 1200$ $\quad$ $L/D = 1$ $\quad$ $n = 10$

$D/t = 1200$	$L/D = 1$	$n = 10$
	$= 2$	$= 7$
	$= 4$	$= 5$
	$= 8$	$= 4$
	$= 12$	$= 3$

It must be kept in mind, however, that this will hold for large deformations only (approximately $d_o/D > .4$ for collapse and $w_o/a > .2$ for buckling).

III. Work, energy and impulse relations

A. Impulse

Using the concept that a structure under impulsive loading of short duration is equivalent to a structure moving with an initial velocity,[3] an expression for the impulse as a function of the absorbed energy of the shell was derived in Reference 1. Assuming that the impulse per unit mass is nonuniform and given by

$$I(x,\phi) = I_o\, g(x,\phi) \qquad [18]$$

the total impulse on the shell can be written as[1]

$$I_t = \int_0^{\ell}\int_0^{2\pi} \sqrt{V \frac{2\rho t}{\int_0^{2\pi}\int_0^{\ell} g^2(x,\phi)\, a\, d\phi\, dx}}\; g(x,\phi)\, a\, d\phi\, dx \qquad [19]$$

This relation would enable us to compute the impulse which results in a given plastic deformation with the help of Figure 2.

B. Energy relation

Keil[4] and others have suggested that orders of magnitude of deflections could be obtained by equating the shock wave energy radiated toward the structure to the energy absorbed in plastically deforming the structure. The energy absorbed by the shell in appropriate failure patterns for typical cases is given by Figure 2. and the equations in the previous section. The energy radiated toward the shell by the blast is not easy to ascertain expecially since the pressure is nonuniform around the shell. However, some approximate relations will be given which are based on some earlier work.[4,9]

Keil[4] gives as the shock wave energy density (i.e. the shock wave energy per unit area of wave front) for a plane shock wave of exponential decay form

$$\bar{E} = \frac{1}{\rho_o c}\, p_o^2\, \theta \qquad [20]$$

where ρ is the density of the medium, θ is the decay time, c is the sound velocity in the medium and p_o is the peak overpressure. Baker and Schuman[9] give values for P_o and θ for bare pherical pentolite explosive in free air. They give 1.1 pound TNT as equivalent of 1 pound pentolite and the weight in free air is equivalent to 1.8 times the weight on the ground. For rough approximation one could write

$$\begin{aligned} (\text{Area of Shell})\bar{C}\bar{E} &= V \\ \text{or} \qquad \bar{C}\pi D L \bar{E} &= V \end{aligned} \tag{21}$$

where $\bar{C}$ is a constant which corrects for nonuniform pressure distribution around the shell and other unknown factors associated with the blast. As a first approximation $\bar{C}$ could be taken equal to unity.

Further study of experiment and theory may enable one to arrive at a better value for $\bar{C}$. From the Baker and Schuman results together with the Keil formula above and the curves given in this report, the plastic deformations could then be estimated for different charge weights and distances.

C. More accurate calculations based on measured pressure data

If the pressure over the shell is known as a function of space and time then the actual impulse applied to the shell can be computed instead of merely the incident and reflected impulse as measured in previous work.[5,6] This impulse will be

$$I_t = \int_0^L \int_0^{2\pi} \int_0^\theta p(z, \phi, t)\, dt\, a\, d\phi\, dz$$

The results can then be compared directly with the calculations based on equation [19]. This will be the critical test of the theory. Ballistic Research Laboratories are now in the process of measuring such pressures.

IV. Discussion

It is seen from Figure 2 that very thin short shells will buckle before they collapse and longer thicker shells will collapse first. It is interesting to note that the energy in collapse is very insensitive to change in shell geometry in the medium deflection region $.2 < d_o/D < .6$. Furthermore the dimensionless collapse energy $\bar{V}$ seems to be dependent only on L/D and independent of D/t; whereas the buckling post failure energy is sensitive to both D/t and L/D.

It may be desirable for some applications that the shell be forced to buckle and for other applications that it may be forced to collapse. The curves given here can be used to design for a given type of failure and for a given post failure deflection under a given impulse.

REFERENCES

1. Joshua E. Greenspon, "Elastic and Plastic Behavior of Cylindrical Shells Under Dynamic Loads Based on Energy Criteria," J G Engineering Research Associates, Contract No. DA-36-034-ORD-3081 RD with Ballistic Research Laboratories, Tech. Rep. No. 3, Feb., 1963.

2. Joshua E. Greenspon, "Collapse, Buckling and Post Failure of Cylindrical Shells," J G Engineering Research Associates, Contract No. DA-36-034-ORD-3081 RD with Ballistic Research Laboratories, Tech. Rep. No. 4, Dec., 1963.

3. T. H. H. Pian, "Dynamic Response of Thin Shell Structures," Plasticity, Proceedings of the Second Symposium on Naval Structural Mechanics, Pergamon Press, 1960, p. 443.

4. A. H. Keil, "The Response of Ships to Underwater Explosions," David Taylor Model Basin Report 1576, November, 1961 (Reprint of paper presented at Annual Meeting of Society of Naval Architects and Marine Engineers, 16-17 November, 1961.

5. William J. Schuman, Jr., "The Response of Cylindrical Shells to External Blast Loading," Ballistic Research Laboratories, Aberdeen Proving Ground, BRL Memorandum Report No. 1461, March, 1963.

6. William J. Schuman, Jr., "The Response of Cylindrical Shells to External Blast Loading, Part II," Ballistic Research Laboratories, Aberdeen Proving Ground, BRL Memorandum Report No. 1560, May, 1964.

7. B. O. Almroth, "Buckling of a Cylindrical Shell Subjected to Nonuniform External Pressure," Journal of Applied Mechanics, Dec., 1962, p. 675.

8. Iliouchine, "Plasticite," (Translated from the original Russian into French), Edition Eyrolles, 1956, p. 98.

9. W. E. Baker and W. J. Schuman, Jr., " Air Blast Data for Correlation with Moving Airfoil Tests," Ballistic Research Laboratories, Aberdeen Proving Ground, Tech. Note No. 1421, August, 1961.

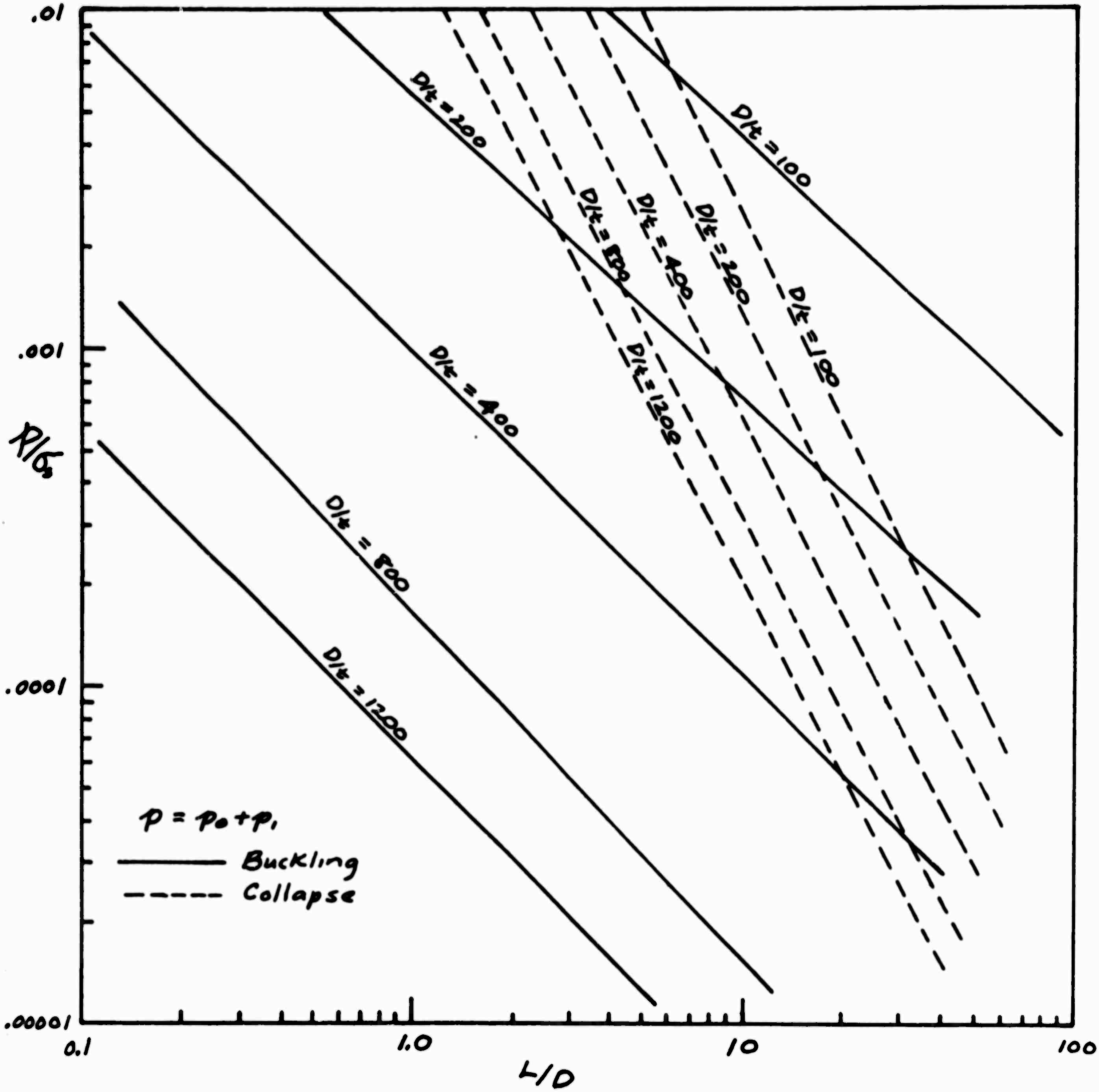

Fig. 1 Buckling and Collapse Loads

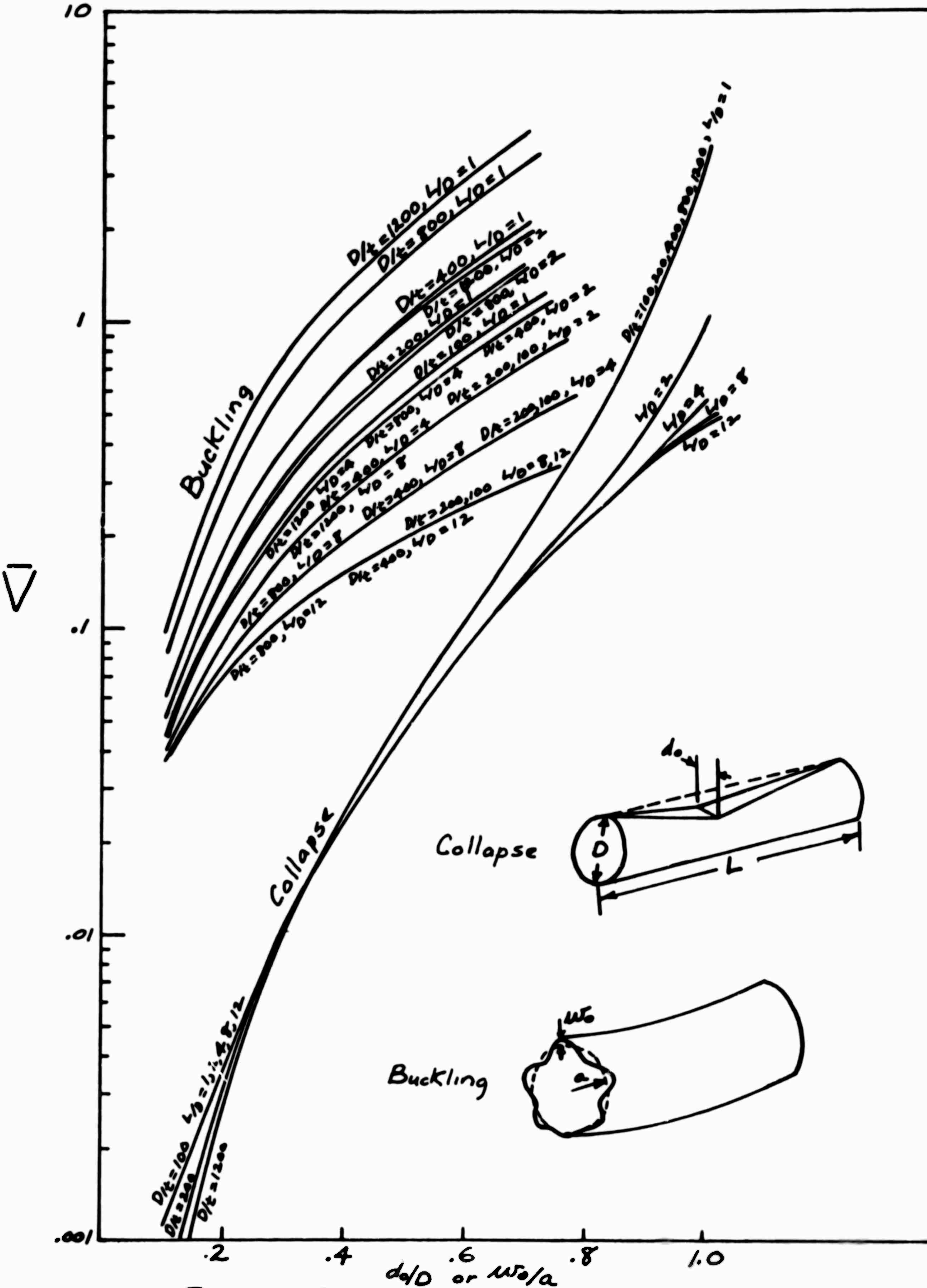

Fig. 2 Post Failure Collapse and Buckling Curves